$$\sigma_z = \frac{p}{K^2-1}$$

σ_θ is given by Equation (13b). Equating σ_θ at r_o to σ_z, we get the surprising result that the limit to $\frac{p}{2S}$ in this case is also given by Equation (35). Thus, the limit curve in Figure 43 has two meanings: it is the limit at which the minimum of the shear stress $\frac{\sigma_\theta - \sigma_r}{2}$ from residual pressures becomes equal to -S at the bore, and it is also the limit at which the bore shear stresses $\frac{\sigma_\theta - \sigma_r}{2}$ and $\frac{\sigma_z - \sigma_r}{2}$ become equal under the bore pressure p.

From the limit curve in Figure 43 and from Equation (35) it is found that

$$\lim_{K \to \infty} \left(\frac{p}{2S}\right) = 1 \tag{36}$$

Thus, the maximum possible pressure in a multiring container designed on the basis of static shear strength using ductile materials is p = 2S. For a ductile material with a tensile yield strength of 2S = 180,000 psi, this means that the maximum pressure is limited to 180,000 psi.

Fatigue Shear Strength Analysis

The optimum design of a multiring container having all rings of the same material and based on fatigue shear strength is found by an analysis similar to that conducted on the basis of static shear strength. Instead of minimizing S in Equation (27), σ given by the fatigue relation, Equation (9) is minimized, i.e.,

$$\frac{\partial\sigma}{\partial k_n} = 0, \; n = 1, 2, \ldots, N-1 \tag{37}$$

The stresses S_r and S_m needed in expressing σ in Equation (9) are given by Equations (16) and (17).

The results of carrying out the analysis are:

$$p_n = p_{n-1} + \frac{p(k_n^2-1)}{4(K^2-1)} \, k_{n+1}^2 \, k_{n+2}^2 \ldots k_N^2 - \frac{\sigma(k_n^2-1)}{2k_n^2}, \; n = 1, 2, \ldots, N-1 \tag{38}$$

$$k_1 = k_2 \ldots = k_N \tag{39}$$

$$\sigma = \frac{5}{2N} \, p \, \frac{K^{2/N}}{K^{2/N}-1} \tag{40}$$

The q_n are again given by Equation (32) and the resulting interference required is

$$\frac{\Delta_n}{r_n} = \frac{5p}{2NE} \tag{41}$$

The result p/σ is plotted in Figure 44. The limit curve is for $S_m = 0$ in the inner cylinder and is given by

$$\lim_{K \to \infty} \left(\frac{p}{\sigma}\right) = \lim_{K \to \infty} \left(\frac{2}{3} \frac{K^2 - 1}{K^2}\right) = \frac{2}{3} \tag{42}$$

If a ductile material has an ultimate tensile strength of 210,000 psi, then Equation (42) gives a maximum pressure of 140,000 psi based upon the shear fatigue criterion.

These results on ductile materials show that higher strength materials will have to be used in order to reach the high pressures desired. Accordingly, an analysis of a multiring container with a high-strength liner is now described.

High-Strength Liner Analysis

The hoop stress σ_θ at the bore of the liner undergoes the greatest range in stress during a cycle of pressure. Therefore, the tensile fatigue criterion is applied to the σ_θ stress. The range in the σ_θ stress at the bore of a multi-ring container depends only upon the over-all ratio K and the bore pressure p and is independent of the number of rings, i.e.,

$$(\sigma_\theta)_r = \frac{p}{2} \frac{K^2 + 1}{K^2 - 1} \tag{43}$$

[Equation (43) is found from Equation (13b) for $r = r_o$, $r_n = r_N$, and $k_n = K$.]

In the formulation of the tensile fatigue criterion the parameter α_r has been defined by Equation (10a). Thus, from Equations (10a) and (43) it is found that

$$\frac{p}{\sigma_1} = 2\alpha_r \frac{K^2 - 1}{K^2 + 1}, \quad \sigma_1 \leqq \sigma_u \tag{44}$$

where σ_u is the ultimate tensile stress of the liner. The ratio p/σ_1 is plotted in Figure 45 for various K and α_r.

The fatigue data at room temperature of high-strength steels ($\sigma_u \leqq 300,000$ psi) listed previously in Tables XLII, XLIII, and XLIV are generally for $\alpha_r \lesssim 0.5$ for lifetimes of 10^4 and greater. Hence, it is concluded that the maximum repeated pressure possible in a multiring container with a liner of $\sigma_u = 300,000$ psi is approximately 300,000 psi if appreciable fatigue life is required. This conclusion presupposes that the outer components can also be designed to withstand the required interface pressure and that sufficient precompression can be provided in the liner so that $\alpha_r = 0.5$ can be expected to give up to 10^4 cycles life. This is investigated next.